PROFORMA FOR SUBMISSION OF INFORMATION AT THE TIME OF SENDING THE FINAL REPORT OF THE WORK DONE ON THE PROJECT

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3.	UGC Approval No. and Date	:	**MH-31/202081/15-16/CRO/212, Dated 20 Sep. 2016**
4.	Date of Implementation	:	**22/09/2016**
5.	Tenure of the Project	:	**2 Years (2016-17, 2017-18)**
6.	Total Grant Allocated	:	**Rs. 1,80,000.00**
7.	Total Grant Received	:	**Rs. 1,35,000.00**
8.	Final Expenditure	:	**Rs. 1,81,926.00**
9.	Title of the Project	:	**"Higher Dimensional Cosmology with Dark Energy Models"**

10. Objectives of the Project:

The Following objectives were set for present investigation:

- Higher Dimension cosmology with Dark Energy
- Higher Dimension Cosmology with Dark Matter
- Dark Energy with Gravity

11. Achievements of the Project: determining the cosmological field equation is still very much debated and led to a wide discussion around different theoretical proposals. A suitable conceptual scheme could be represented by gravity models that naturally generalize Einstein theory like higher order gravity theories and higher dimensional ones. Both of these two different approaches allow one to define, at the effective level ,Einstein field equation s equipped with source –like energy –momentum tensor of geometrical origin. The possibility is discussed to develop a five dimensional fourth order gravity model whose lower –dimensional reduction could provide an interpretation of cosmological four-dimensional matter energy components. We describe the basic concept of the model, the complete field equation formalism and the 5-D to 4-D reduction procedure. Five dimensional f(R) field equation turn out to be equivalent, on the four dimensional higher surface orthogonal to the extra co-ordinate, to an Einstein –like cosmological model with higher derivative and higher –dimensional counter –term .The effective four-dimensional picture and the behaviour of the geometrically introduced sourced are finally outlined in correspondence to simple cases of such higher dimensional solutions.

12. Summary of the Findings: The recent discovery of dark energy has prompted an investigation of ways in which the accelerated expansion of the universe can be realized. In this dissertation, we present two separate projects related to dark energy. The first project analyzes a class of brane world models in which multiple branes float in a five-dimensional anti-de Sitter bulk, while the second investigates a class of dark energy models from an effective field theory perspective. Investigations of models including extra dimensions have led to modifications of gravity involving a number of interesting features. In particular, the Randall-Sundrum model is well-known for achieving an amelioration of the hierarchy problem. However, the basic model relies on Minkowski branes and is subject to solar system constraints in the absence of a radion stabilization mechanism. We present a method by which a four-dimensional low-energy

description can be obtained for brane world scenarios, allowing for a number of generalizations to the original models. This method is applied to orbifolded and uncompactified N-brane models, deriving an effective four-dimensional action. The parameter space of this theory is constrained using observational evidence, and it is found that the generalizations do not weaken solar system constraints on the original model. Furthermore, we find that general N-brane systems are qualitatively similar to the two-brane case, and do not naturally lead to a viable dark energy model. We next Investigate dark energy models using effective field theory techniques. We describe dark energy through a quintessence field, employing a derivative expansion. To the accuracy of the model, we find transformations to write the description in a form involving no higher-order derivatives in the equations of motion.

13. Contribution to the Society: Physical cosmology is studied by scientists such as astronomers and physicists, as well as philosophers such as metaphysicians ,philosophers of physics and philosophers of space and time. Because of this shared space with philosophy ,theories in physical cosmology may include both scientific and non-scientific propositions, and may depend upon assumptions that cannot be tested. Cosmology differs from astronomy in that the former is concerned with the universe as a whole while the latter deals with individual celestial objects. Modern physical cosmology is dominated by the Big-Bang theory, which attempts to bring together observational astronomy and particle physics, more specifically ,a standard parameterization of the Big-bang with dark matter and dark energy.We investigate a cosmological model resulting from a dimensional reduction of the higher dimensional dRGT massive gravity. By using the Kaluza-Klein dimensional reduction, we obtain an effective four-dimensional massive gravity theory with a scalar field. It is found that the resulting theory corresponds to a combined description of mass varying massive gravity and quasidilation massive gravity. By analyzing the cosmological solution, we found that it is possible to obtain the late-time expansion of the universe due to the graviton mass. By using a dynamical system approach, we found regions of model parameters for which the late-time expansion of the universe is a stable fixed point. Moreover, this also provides a mechanism to stabilize the extra dimensions. Cosmic evolution does have useful practical application from which application from which humanity could benefit.

14. **Weather any Ph.D. Enrolled/ Produced out of the Project: No**

15. **No. of Publications out of the Project: 01**

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Cosmological Model of the Universe with Ω_M-Dependent Cosmological Constant in Higher Dimensional Space Time

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This paper deals with the evolution of the scale factor with time varying c by assuming λ-tensor is proportional to the stress energy tensor $T = T_i^i$ which has the dimension of energy density and is Lorentz invariant. We also derived analytical expression for the look-back time, proper distance, luminosity distance and angular diameter in the framework of higher dimensional space time.

Kaluza Klein theory,Varying speed of light theory, cosmology, cosmological constant.

PACS number(s): 04.50.Cd; 98.80.-k; 98.80.Es; 98.80.Cq

In a cosmological setting, the variations in c have been shown to solve the cosmological puzzles– the horizon, flatness and Lambda problems of big-bang cosmology. The variations of velocity of light can also solve the quasi-lambda problems. The changing c does not affect the geometry of the universe. In classical electromagnetism the speed of light is only Lorentz invariance and the conservation of energy. One can allow the changing c to do the job normally done by superluminal expansion. For changing c, the gravitational laws should be modified. The basic assumption is that a variable c does not induce corrections to curvature in the cosmological frame and therefore, Einstein's equations, relating curvature to stress energy, are still valid. The reason behind this postulate is that c changes in the local Lorentzian frames associated with cosmological expansion. The effect is a special relativistic effect and not a gravitational effect. Therefore, curvature should not be related with the variations of c. But Moffat[1], Albrecht and Magueijo[2], Barrow[3], Avelino and Martins [4] have conjectured that if the speed of light falls at certain rate then the horizon flatness and monopole problems of the standard can be solved without recourse to inflationary paradism. The Machin varying speed of light (VSL) scenario in which $c = c_0 t^a$ introduced by Barrow [5] has significant advantages to the phase transition scenario in which the speed of light changes suddenly from c^- to c^+, preferred by Albrecht and Magueijo [2].

Kaluza-Klein achievements is shown that five-dimensional general relativity contains both Einstein's four-dimensional theory of F and Maxwell's theory electromagnetism. A number of authors [Sahdev [6], Emelyanov et al.[7], Chatterjee and Bhui [8,9], Wesson [10], Chatterjee et al.[11,12], Chakraborty and Roy [13]] have studied physics of the Universe in higher-dimensional space-time. Overduin and Wesson [14] have presented an excellent review of higher-dimensional unified theories, in which the cosmological and astrophysical implications of extra-dimension have been discussed. Also Khadekar and Wanjari [15], Khadekar and Shelote [16] studied behavior of strange quark matter in the context of Kaluza-Klein theory of gravitations.

As it is well known, the Einstein field equations with a non-zero λ can be rearranged so that their right-hand sides consist of two terms: the stress-energy tensor of the ordinary matter and an additional λ-tensor. A quick look at the λ-tensor T_{ij}^{ν} shows that it consists of two parts: the λ-part, having the dimension of the energy density, which is multiplied by the components of the metric tensor forming its metric part:

$$T_{ij}^{(\nu)} = \left(\frac{c^4\lambda}{8\pi G}\right) g_{ij} = \Lambda g_{ij}, \tag{1}$$

where Λ satisfies the requirements: (i) it should have the dimension of energy density, and (ii) it should be invariant under Lorentz transformation. It is also identified with vacuum energy because this quantity satisfies the requirements asked of Λ.

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Instead of identified Λ with the vacuum energy Majernik [17] has identified Λ with stress energy scalar $T = T_i^i$ which has the dimension of energy density and is Lorentz Invariant. Majernik [18] make the ansatz

$$\Lambda_A = \frac{c^4\lambda_A}{8\pi G} = \kappa T_i^i = \kappa T, \tag{2}$$

or

$$\lambda_A = \frac{8\pi G\kappa T}{c^4}, \tag{3}$$

where κ is a dimensionless constant to be determined. Λ_A is a dynamical quantity, changing over time, representing, in the quintessence theory, the quintessence component.
In this study we describe higher dimensional cosmological model with varying c in which we consider Eq. (2) as a cosmological constant. We investigate the evolution of the scale factor in which the cosmological constant is given by the scalar arisen by the contraction of the stress energy tensor in the framework of Kaluza-Klein theory of gravitation with varying speed of light c.
The paper is organize as follows: section 2, deals with the Einstein's field equations with energy momentum tensor for a perfect fluid and determine Λ_A in terms of Ω_M in the framework of Kaluza-Klein theory of gravitation with varying speed of light c. In section 3, we investigate the evolution of the scale factor in the dependence on Ω_M in the matter dominated universe by choosing $p = 0$ i.e. $T = \rho_{_M}c^2$. In section 4, we have analyzed the cosmological test pertaining to look back time, proper distance, luminosity distance and angular diameter distance for Kaluza-Klein type cosmological model of the Universe with varying c. Concluding remarks are given in section 5.

1 Varying c cosmological model with a Ω_M-dependent cosmological constant

We consider Kaluza-Klein type cosmological model with time dependent c of the form

$$ds^2 = c^2(t)dt^2 - R^2(t)\times$$

$$\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2) + (1-Kr^2)d\psi^2\right], \tag{4}$$

where $R(t)$ is a scale factor.
The Einstein field equations can be written in the form

$$R_{ij} - g_{ij}\frac{1}{2}R = \frac{8\pi G}{c^4}(T_{ij}^{(m)} + T_{ij}^{(\nu)}), \tag{5}$$

where $T_{ij}^{(m)}$ is the energy-momentum tensor for the perfect fluid

$$T_{ij}^{(m)} = (\rho c^2 + p)u_iu_j - pg_{ij}, \tag{6}$$

and

$$T_{ij}^{(\nu)} = g_{ij}\Lambda, \quad \Lambda = \frac{\lambda c^4}{8\pi G}. \tag{7}$$

Putting $\Lambda = \Lambda_A = \kappa T$ we have $T_{ij}^{(\nu)} = g_{ij}\kappa T$.
By using Eq. (7) and (6), in the field equations (5), we have

$$R_{ij} - g_{ij}\frac{1}{2}R =$$

$$\frac{8\pi G}{c^4}[(\rho c^2 + p)u_iu_j - (p - \kappa T)g_{ij}]. \tag{8}$$

The stress-energy tensor of the cosmic medium T_j^i has only five non-zero components $T_0^0 = \rho c^2, T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p$.
Therefore,

$$T = \rho c^2 - 4p. \tag{9}$$

By using Eq. (9) into Eq. (8) we get

$$T_{ij}^{(m)} + T_{ij}^{(\nu)} =$$

$$(\rho c^2 + p)u_iu_j - [p(1+4\kappa) - \kappa\rho c^2]g_{ij}. \tag{10}$$

This is the energy-momentum tensor for a perfect fluid with effective density $\hat{\rho}$ and pressure $\hat{p}$,

$$T_{ij}^{(m)} + T_{ij}^{(\nu)} = (\hat{\rho}c^2 + \hat{p})u_i u_j - \hat{p}g_{ij}. \tag{11}$$

The quantities $\hat{\rho}$ and $\hat{p}$ can be determined by the equation of state $p = \omega\rho c^2$ in the form of

$$\hat{\rho} = \rho[1 + \kappa(1 - 4\omega)]$$

and

$$\hat{p} = \rho c^2[\omega - \kappa(1 - 4\omega)]. \tag{12}$$

Next we will find the evolution of scale factor in the radiation-dominated and matter-dominated eras.
By considering the line element (4) with the help of the Einstein's field equations (5) and energy momentum tensor (6) and the non-zero cosmological term λ acquire the following forms:

$$6\frac{\dot{R(t)}^2}{R(t)2} = 8\pi G\rho + \lambda c^2 - 6\frac{Kc^2}{R^2(t)}, \tag{13}$$

and

$$\ddot{R}(t) =$$

$$\frac{4\pi G}{3}\left(-\rho - \frac{2p}{c^2}\right) + \frac{\dot{R}\dot{c}}{c} + \frac{\lambda c^2}{6}R(t), \tag{14}$$

After eliminating ρ by using Eqs. (13) and (14) with the help of equation of state $p = \omega\rho c^2$ we get

$$\frac{\ddot{R}}{R} + (1 + 2\omega)\left(\frac{\dot{R}^2}{R^2} + \frac{Kc^2}{R^2}\right)$$

$$-\frac{\dot{R}\dot{c}}{c} - \frac{(1+\omega)}{3}\lambda c^2 = 0. \tag{15}$$

Now we discuss the evolution of the scale factor in the matter-dominated epoch. We have determine the values of dimensionless constant κ by assuming the Universe is flat i.e.($\Omega_{tot} = 1$). Since $\Omega_M = \frac{\rho_M}{\rho_{crit}} < 1$, we suppose that the remaining energy of cosmological constant required to produce a geometrical flat Universe is given by the equation [Majernik (2003)]

$$\Omega_M + \Omega_\Lambda = \Omega_M + \kappa\,\Omega_M = 1.$$

This gives

$$\kappa = \frac{1}{\Omega_M} - 1. \tag{16}$$

By using Eq. (16) into Eq. (2) we get

$$\Lambda_A = \left(\frac{1}{\Omega_M} - 1\right)T = \left(\frac{1}{\Omega_M} - 1\right)(\rho c^2 - 4p). \tag{17}$$

In the radiation-dominated era with $\omega = \frac{1}{4}$ the cosmological evolution is described by the standard Kaluza-Klein model with zero cosmological constant. However, in the matter-dominated era with $\omega = 0$ in Eq. (12) we get the following effective density and pressure

$$\widehat{\rho} = \rho + \kappa\rho, \quad \widehat{p} = -\kappa\rho c^2.$$

For the radiation-dominated era is described by standard Kaluza-Klein type cosmological model, we get $T = 0$, so we will not further deal with it, instead we will investigated the evolution of the scale factor in dependence on Ω_M in the matter-dominated Universe.

2 Matter-dominated era

In this section we find the evolution of the scale factor in dependence on Ω_M in the matter dominated Universe i.e. $p = 0$.
By using $T = \rho_M c^2$ into the Eq. (2), we get

$$\Lambda_A = \kappa\rho_M c^2 = \left(\frac{1}{\Omega_M} - 1\right)\rho_M c^2 =$$

$$(\rho_{crit} - \rho_M)c^2 = \rho_{crit}(1 - \Omega_M)c^2. \tag{18}$$

We obtain the critical density ρ_{crit} by means of Eqs. (18) and (13)

$$8\pi G\rho_{crit} = \frac{6\dot{R}^2}{R^2}. \tag{19}$$

From Eq. (19) with Eq. (17) we get the equation for the evolution of $R(t)$ for the matter dominated era

$$\ddot{R} = (1 - 2\Omega_M)\frac{(\dot{R}(t))^2}{R(t)} + \frac{\dot{R}\dot{c}}{c}. \tag{20}$$

To solve this equation substitute [Barrow [5]] $c = c_0 t^a$, $(a > 0)$ in Eq. (20), we get

$$\ddot{R} = (1 - 2\Omega_M)\frac{(\dot{R}(t))^2}{R(t)} + \frac{a\dot{R}}{t}. \tag{21}$$

The solution can be found for an arbitrary time function $\Omega_M(t)$.
For $R = e^y$, we have

$$\dot{R} = \dot{y}e^y, \quad \ddot{R} = (\ddot{y} + \dot{y}^2)e^y,$$

after putting this value in Eq. (21) we get

$$-2\Omega_M(\dot{y})^2 + \frac{a\dot{y}}{t} = \ddot{y}.$$

By inserting $\dot{y} = q$, above equation becomes

$$\frac{[aqt^{(a-1)} - \dot{q}t^a]}{q^2} = 2\Omega_M t^a$$

its solution can be written as

$$q = \frac{t^a}{\int 2\Omega_M t^a dt + C_1}.$$

Since $\dot{y} = q$ we have

$$y = \int\left(\frac{t^a}{\int(2\Omega_M t^a)dt + C_1}\right)dt + C_2.$$

By putting this value of $y(t)$ in the relation $R = e^y$, we get

$$R(t) = exp\int\left[\frac{t^a}{\int(2\Omega_M t^a)dt + C_1}\right]dt + C_2, \tag{22}$$

where C_1 and C_2 are the integration constants.
We assume that Ω_M does not change during the matter-dominated era.
After solving Eq. (22), we get

$$R(t) = (2\Omega_M)^{\frac{1}{(2\Omega_M)}}\left[\frac{C_2 t^{(a+1)}}{(a+1)} + C_1 C_2\right]^{\frac{1}{(2\Omega_M)}}. \tag{23}$$

Next we have to specify the Ω_M and the boundary conditions for differential Eq. (22).
As the boundary condition we set $R(t = 0) = 0$. Inserting $\Omega_M = \frac{1}{4}$ into Eq. (23), the evolution scale factor $R(t)$ take the form

$$R(t) = C_0^2 t^{2(a+1)}, \tag{24}$$

where $C_0 = \frac{C_2}{2(a+1)}$.
The scale factor (24) implies a model of the Universe having following properties:
(i) The Hubble parameter $H = \frac{\dot{R}}{R} = \frac{2(a+1)}{t}$ which implies that the age of this universe t_0 is approximately 4.10^{10} Yr. In the cosmological model with Λ_A, the Universe is old enough for the evolution of globular clusters.
(ii) In our model, the decelerator parameter q_0 has the value $q_0 = -\frac{(2a+1)}{2(a+1)}$, i.e. the Universe is accelerated in concord with the recent observations.
(iii) The cosmological constant is time- dependent $\lambda_A = \frac{18(a+1)^2}{c^2t^2}$.

2.1 Evolution of $R(t)$ for the different density parameters

In this section we present the solutions of Eq. (21) for some selected constant values of Ω_M and then derive the cosmological parameters of the corresponding models of universe.
(A) For $\Omega_M = 0$, we obtain a typical inflationary solution of Eq. (21),

$$R(t) = e^{\left[C_1\left(\frac{t^{(a+1)}}{(a+1)} - C_2\right)\right]} = R_0 e^{[C_0 t^{(a+1)}]}, \tag{25}$$

where $C_0 = \frac{C_1}{(a+1)}$.
For early Universe, it is reasonable to assume that at the very beginning of the cosmic evolution, the radius of the Universe was of the order of the Planck length, therefore we put in Eq. (25) the integration constants R_0 and C_0 equal to l_p and $\frac{1}{l_p}$, respectively. Then, we obtain for the initial radius and the velocity the values

$$R(0) = l_p = 4.3 \times 10^{-35} m,$$

$$\dot{R(0)} = \frac{l_p}{t_p} = 3 \times 10^8 m s^{-1},$$

respectively.
(B) For $\Omega_M = \frac{1}{2}$, the evolution of R(t) is

$$R(t) = C_0 t^{(a+1)} + C_2,$$

where C_1 is a constant. With $C_2 = 0$, this gives

$$R(t) = C_0 t^{(a+1)},\ H = \frac{(a+1)}{t},\ \ q_0 = -\frac{a}{(a+1)}.$$

In the following section we derived the analytical expression the look back time, proper distance, luminosity distance and angular distance for the value of $R \propto t^{2(a+1)}$.

3 Kinematics tests

The kinematical relation distances must be confronted with the observations in order to put limits on the free parameters of the model. The formulas for $R(t)$ derived in Eqs. (24) i.e. $R \propto t^{2(a+1)}$ may be used to extend phenomenological analysis to arbitrary large redshifts.

3.1 Look-Back Time

The look-back time, $\Delta t = t_0 - t(z)$, is the difference between the age of the Universe at present time (z=0) and the age of the Universe when a particular light ray at redshift z was emitted. The radiation travel time for a photon emitted by a source at instant t and received at t_0 is,

$$\Delta t = t - t_0 = \int_R^{R0} \frac{dR}{\dot{R}}\ , \tag{26}$$

where R_0 is the present scale factor of the Universe. For a given redshift z, the cosmic scale factor of the Universe $R(t_z)$ is related to R_0 by $1 + z = \frac{R_0}{R}$, where $R = R_0$ at the present epoch. An object at $z = 1$ emitted its light when the Universe was half its present scale ($R = 0.5R_0$). How long ago the light was emitted (look back time $\Delta t = t_0 - t(z)$) depends on the dynamics of the Universe. Therefore from Eq. (24), we get

$$1 + z = \left(\frac{t_0}{t}\right)^{2(a+1)}\ . \tag{27}$$

The above equation gives,

$$t = t_0(1+z)^{-\frac{1}{2(a+1)}}. \tag{28}$$

This equation can also be expressed as

$$H_0(t_0 - t) = 2(a+1)[1-(1+z)^{-\frac{1}{2(a+1)}}]. \tag{29}$$

For small z, Eq. (29) gives

$$H_0(t_0 - t) = z - \frac{(2a+3)}{4(a+1)}z^2 + \cdots. \tag{30}$$

By using Eq. (24) the deceleration parameter can be written as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{(2a+1)}{4(a+1)}. \tag{31}$$

Using Eq. (31) in Eq. (30), we get

$$H_0(t_0 - t) = z - \frac{(q+2)}{2}z^2 + \cdots. \tag{32}$$

Taking the limit $z \to \infty$ in Eq. (29) the present age of the Universe is

$$t_0 = 2(a+1)H_0^{-1} = -\frac{(2a+1)}{q}H_0^{-1}. \tag{33}$$

3.2 Neoclassical Tests (proper distance)

A photon emitted by a source with coordinate $r = r_1$ and $t = t_1$ and received at a time t_0 by an observer located at $r = 0$ will follow a null geodesic with (θ, ϕ) = constant.
The proper distance $d(z)$ between the source and the observer is given by

$$d(z) = R_0 \int_R^{R_0} \frac{dR}{R\dot{R}} \quad . \tag{34}$$

The radial coordinate distance as function of redshift is given by

$$r_1(z) = \int_{t_1}^{t_0} \frac{dt}{R} =$$

$$-\frac{2(a+1)}{(2a+1)}H_0^{-1}R_0^{-1}\left[1-(1+z)^{\frac{(2a+1)}{2(a+1)}}\right]. \tag{35}$$

Hence, proper distance as a function of redshift is given by,

$$d(z) = r_1 R_0 =$$

$$-\frac{2(a+1)}{(2a+1)}H_0^{-1}\left[1-(1+z)^{\frac{(2a+1)}{2(a+1)}}\right]. \tag{36}$$

For small z Eq. (36) becomes,

$$H_0 d(z) = z - \frac{1}{4(a+1)}z^2 + \cdots. \tag{37}$$

By using Eq. (31) in above equation, we get

$$H_0 d(z) = z - \frac{(q+1)}{2}z^2 + \cdots\cdots. \tag{38}$$

From Eq. (36), it is observe that the distance $d(z)$ is maximum at $z \to \infty$. Hence

$$d(z=\infty) = -\frac{2(a+1)}{(2a+1)}H_0^{-1}. \tag{39}$$

3.3 Luminosity Distance

The luminosity distance of a light source is defined as the ratio of the detected energy flux L, and the apparent luminosity l, i.e. $d_L^2 = \frac{L}{4\pi l}$.
For the Kaluza-Klein metric (4), it takes the form

$$d_L = R_0\, r_1(z)(1+z) = d_z(1+z), \tag{40}$$

where $r_1(z)$ is the radial coordinate distance of the object at light emission.
By using Eq. (35), one gets,

$$H_0 d_L = -\frac{2(a+1)}{(2a+1)}(1+z)\left[1-(1+z)^{\frac{(2a+1)}{2(a+1)}}\right]. \tag{41}$$

For small z, after some algebra and by using Eq. (31), Eq. (41) gives

$$H_0 d_L = z - \frac{(q-1)}{2}z^2 + \cdots. \tag{42}$$

We also note that for the same redshift the luminosity distance is larger for lower values of q. Thus, for $q = 1$, we have

$$d_L = \frac{z}{H_0}, \tag{43}$$

and for $q = 0$, we get

$$d_L = \frac{1}{H_0}\left(z + \frac{1}{2}z^2\right). \tag{44}$$

3.4 Angular Diameter Distance

The angular diameter distance d_A is defined as the ratio of an object's physical transverse size to its angular size. The angular diameter d_A of a light source of proper distance $d(z)$ is given by

$$d_A = d(z)(1+z)^{-1} = d_L(1+z)^{-2}. \tag{45}$$

By using Eq. (41), one gets

$$H_0 d_A = \frac{2(a+1)}{(2a+1)}\left[\frac{(1+z)^{\frac{(2a+1)}{2(a+1)}}-1}{(1+z)}\right]. \tag{46}$$

Usually d_A has a minimum (or maximum) for some $z = z_c$. The angular diameter distance effect means the light is spread over a large angular area.

4 Conclusion

In this paper we have consider a minimalist varying-c theory proposed by Albrecht and Magueijo (1999) as a new way of solving higher-dimensional problems. The idea of varying speed of light seriously as a physical effect that might have happened in the very early Universe and today confined to a very narrow range admissible by inaccuracy of existing bounds on variability of c. We have discussed the evolution of the scale factor for cosmological model with Ω_M-dependent cosmological constant, which is given by the scalar arising from the contraction of stress energy tensor in the framework of higher-dimensional VSL theory. By considering the flat universe with varying c determined the values of Λ_A in terms Ω_M in Eq. (18). No coincidence problem exists in our model of the Universe because Λ_A is functionally connected with Ω_M in such a way that this ratio in the matter dominated epoch does not vary over time. It is also observed from Eq. (9), $T = 0$ that for the radiation dominated epoch $\omega = \frac{1}{4}$.
In section 3 we have consider the flat and matter dominated Universe and obtained evolution of the equation (21) for $c = t^a$. This equation describe the time dependence of the scale factor as a function of $\Omega_M(t)$. In the same section, for different value of $\Omega_M(t)$ we have obtained different cosmological model, i.e. the scale factor evolves in different ways and shown that the density parameter Ω_M as the ratio of $\rho_{_M}$ and ρ_{crit} may be finite although both quantities are infinite. This Universe is strictly flat, is accelerating, is old enough and is causally connected.
In the last section, for the value of R in Eq. (24) we have discussed the well known astrological phemenon namely look back time, proper distance, luminosity distance and angular diameter distance in the framework of Kaluza-Klein theory of gravitation.

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1 Moffat J W 1993 Superluminary universe: a possible solution to the initial value problem in cosmology *Int. J. Mod. Phys.* D **2** 351
2 Albrecht A and Magueijo J 1999 Time varying speed of light as a solution to cosmological puzzles *Phys. Rev.* D **59** 043516
3 Barrow J D 1999 Cosmologies with varying light speed *Phys. Rev.* D **59** 043515
4 Avelino P P and Martins C J A P 1999 Does a varying speed of light solve the cosmological problems? *Phys. Lett.* B **459** 2741
5 Barrow J D 2003 Unusual features of varying speed of light cosmologies *Phys. Lett.* B **564** 1
6 Sahdev D 1984 Perfect-fluid higher-dimensional cosmologies *Phys. Rev.* D **30** 2495
7 Emelyanov V M, Nikitin Yu P, Rozental J L and Berkov A V 1986 Physics in multidimensional spaces and the beginning of metagalaxy *Phys. Rep.* **143** 1
8 Chatterjee S and Bhui B 1990 Homogeneous cosmological model in higher dimension *Mon. Not. R. Astron. Soc.* **247** 57
9 Chatterjee S and Bhui B 1993 Spherical symmetry and mass-energy in higher dimensions *Int. J. Theor. Phys.* **32** 671
10 Wesson P S 1992 A physical interpretation of Kaluza-Klein cosmology *Astrophys. J.* **394** 19
11 Chatterjee S, Panigrahi D and Banerjee 1994 A Inhomogeneous Kaluza-Klein cosmology *Class. Quantum Grav.* **11** 371
12 Chatterjee S, Bhui B, Basu M B and Banerjee A 1994 Inhomogeneous model with a cosmological model *Phys. Rev.* D **50** 2924
13 Chakraborty S and Roy A 1999 String-dust in higher-dimensional cylindrically symmetric cosmological model *Int. J. Mod. Phys.* D **8** 645
14 Overduin J M and Wesson P 1997 Kaluza-Klein gravity *Phys. Rep.* **283** 303
15 Khadekar G S and Wanjar R 2009 Domain wall with strange quark matter in Kaluza-Klein type cosmological model *Int. J. Theor. Phys.* **48** 2550
16 Khadekar G S and Shelote R 2012 Higher dimensional cosmological model with quark and strange quark matter *Int. J. Theor. Phys.* **51** 1442
17 Majernik V 2001 The cosmic acceleration due to an alternative interpretation of the cosmological constant *Phys. Lett.* A **282** 362
18 Majernik V 2003 Cosmological model with Ω_M-dependent cosmological constant *Gen. Rel. and Gravit.* **35** 6 1007