ED-310

M.A./M.Sc 1st Semester
Examination, March-April 2021

MATHEMATICS

Paper - II

Real Analysis-I

Time : Three Hours] [*Maximum Marks* : 80
[*Minimum Pass Marks* : 16

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove the Dirichlet's tests for uniform convergence.

(*b*) State and prove the Cauchy's criterion for uniform convergence.

(*c*) (*i*) Test for uniform convergence of the series :

$$\sum_{n=0}^{\infty} a^n \cos nx = 1 + a\cos x + a^2 \cos 2x + \ldots + a^n \cos nx + \ldots$$

(*ii*) Prove that if δ is any fixed positive number less than unity, the series $\sum_{n=1}^{\infty} \frac{x^n}{n+1}$ is uniformly convergent in $[-\delta, \delta]$

Unit-II

2. (*a*) State and prove the Tauber's theorem.

(*b*) Prove that the sum of an absolute convergent series does not alter with any rearrangement of terms.

(*c*) (*i*) Prove that the series

$$1+\frac{1}{2}-1+\frac{1}{3}+\frac{1}{4}-\frac{1}{2}+.....+ \text{ is } \log 2.$$

(*ii*) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{a^n}{\sqrt{n+1}}$.

Unit-III

3. (*a*) Let Ω be the set of all invertible linear operators on R^n.

(*i*) If $A \in \Omega$, $B \in L(R^n)$ and $\|B-A\| \, \|A^{-1}\| < 1$. Then prove that $B \in \Omega$.

(*ii*) Ω is open subset is $L(R^n)$ and the mapping $f : \Omega \to \Omega$ defined by $f(A) = A^{-1}$ for all $A \in \Omega$ is continuous.

(*b*) State and prove tha chain rule.

(*c*) Write short note on derivatives is an open subset of R^n.

Unit-IV

4. (*a*) If $u = \dfrac{x+y}{2}, v = \dfrac{y+z}{x}, w = \dfrac{y(x+y+z)}{x^2}$,

show that *u*, *v*, *w* are not independent and find the relations among them.

(*b*) Prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} \times \frac{\partial(x,y,z)}{\partial(u,v,w)} = 1 = JJ'$$

(*c*) Determine the maximum and minimum values of the function

$$f(x,y) = x^2 + y^2 + \frac{3\sqrt{3}}{2}xy$$

subject to the constraint $4x^2 + y^2 = 1$.

Unit-V

5. (*a*) Write definition of :

(*i*) The integral of 1-form

(*ii*) The integral of 2-form

(*iii*) The Triple integral

(*b*) State and prove the partitions of unity.

(*c*) Lew w and λ be k and m-forms respectively of class $\mathscr{C}$ in some open set $E \subset R^n$. Then prove that

$$d(w \wedge \lambda) = (dw) \wedge \lambda + (-1)^k w \wedge d\lambda.$$