

# DD-2863

## B. C. A. (Part II) EXAMINATION, 2020

*Paper Second*

### DIFFERENTIATION AND INTEGRATION

*Time : Three Hours*

*Maximum Marks : 50*

Note : All questions are compulsory. Attempt any two parts from each question. All questions carry equal marks.

#### Unit—I

1. (a) If:

$$y = \sin (m \sin^{-1} x),$$

then prove that :

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

(b) Find 'C' of mean value theorem, if:

$$f(x) = x^3 - 5x^2 - 3x, a = 1, b = 3$$

(c) Expand  $\tan^{-1}x$  in powers of  $\left(x - \frac{\pi}{4}\right)$ .

#### Unit—II

2. (a) Find the asymptotes of the curve :

$$y^3 + x^2y + 2xy^2 - y + 1 = 0.$$

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- (b) Prove that for
- $s = ae^{x/c}$
- :

$$c\rho = s\sqrt{s^2 - c^2}$$

- (c) Trace the curve :

$$y^2(2a-x) = x^3.$$

Unit—III

3. (a) If
- $u = e^{xyz}$
- , show that :

$$\frac{\partial^2 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}.$$

- (b) Find the directional derivative of :

$$\phi = xy + yz + zx$$

in the direction of the vector  $i + 2j + 2k$  at the point  $(1, 2, 0)$ .

- (c) If
- $x = r \sin \theta \cos \phi$
- ,
- $y = r \sin \theta \sin \phi$
- ,
- $z = r$
- , then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta.$$

Unit—IV

4. (a) Evaluate :

$$\int \frac{1}{x^3 - 1} dx.$$

- (b) Prove that :

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}.$$

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- (c) Evaluate :

$$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx.$$

Unit—V

5. (a) Evaluate :

$$\int_0^1 \int_0^{1-x^2} \int_0^{x+y^2} x dx dy dz.$$

- (b) Change the order of an integration :

$$\int_0^a \int_{x^2/a}^{2a-x} f(x, y) dx dy.$$

- (c) Find the complete area of the circle :

$$x^2 + y^2 = a^2.$$

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